18.07.15

Probit Modell

$y_1 \sim B(\pi_1) \qquad \pi_1 = \Phi(\eta_1)$

$y_1^* = \eta_1 + \varepsilon_1 \qquad \varepsilon_1 \sim N(0,1)$

Definire

$y_1 = 1 \quad \text{if} \quad y_1^* > 0$

$y_1 = 0 \quad \text{if} \quad y_1^* \leq 0$

Falls bivariate (angenommen im Weiteren Gausscopula)

$y_2 \sim F_{\theta_2}$

vom 06.07.15 wissen wir

$$F_{y_1^*|y_2^*}(u_1) = \Phi\left(\frac{\Phi^{-1}(u_1) - \rho\,\Phi^{-1}(u_2)}{\sqrt{1-\rho^2}}\right) = x = \frac{\partial C(u_1,u_2)}{\partial u_2}$$

$$\Rightarrow y_1^* = \Phi^{-1}(x)\sqrt{1-\rho^2} + \rho\,\Phi^{-1}(u_2) + \mu_1$$

$$\Rightarrow y_1^* \sim N\left(\rho\,\Phi^{-1}(u_2) + \mu_1, 1-\rho^2\right)$$

FC's of latent Variables (Univariate) 18.07.15

$$p(y_1^* | \eta_1, y_1) \propto p(y_1^*, y_1, \eta_1)\, p(y_1, \eta_1)$$

$$\propto p(y_1 | \eta_1, y_1^*)\, p(y_1^*, \eta_1)$$

$$\propto p(y_1 | y_1^*)\, p(y_1^* | \eta_1)$$

$$\Rightarrow p(y_1^* | \eta_1, y_1 = 1) \propto \mathbb{1}(y_1^* > 0)\, \varphi(y_1^* - \eta_1)$$

$$= \frac{\varphi(y_1^* - \eta_1)}{P(y_1^* \leq 0)}$$

$$= \left.\frac{\varphi(y_1^* - \eta_1)}{\Phi(-\eta_1)}\right\} \text{density of } TN_{(0,\infty)}(\eta_1, 1)$$

entsprechend

$$p(y_1^* | \eta_1, y_1 = 0) = \left.\frac{\varphi(y_1^* - \eta_1)}{1 - \Phi(-\eta_1)}\right\} \text{density of } TN_{(-\infty,0)}(\eta_1, 1)$$

18.03.15

mir Copula:

$$p(u_1^* \mid \eta_1, u_1, u_2) \propto p(u_1, u_1^*, \eta_1, u_2)\, p(\eta_1, u_1, u_2)$$
$$\propto p(u_1 \mid \eta_1, u_1^*, u_2)\, p(u_1^*, \eta_1, u_2)$$
$$\propto p(u_1 \mid u_1^*) \cdot \varphi\left(\frac{u_1^* - (\eta_1 + \rho\, \Phi^{-1}(u_2))}{\sqrt{1-\rho^2}}\right)$$

$\Rightarrow$ wie vorher Truncated normal distribution
for FC's of u_1^*

$$p(u_1^* \mid \eta_1, u_1 = 1, u_2) = \frac{\varphi\left(\frac{u_1^* - (\eta_1 + \rho\, \Phi^{-1}(u_2))}{\sqrt{1-\rho^2}}\right)}{\Phi\left(-\frac{(\eta_1 + \rho\, \Phi^{-1}(u_2))}{\sqrt{1-\rho^2}}\right)}$$

$$p(u_1^* \mid \eta_1, u_1 = 0, u_2) = \frac{\varphi\left(\frac{u_1^* - (\eta_1 + \rho\, \Phi^{-1}(u_2))}{\sqrt{1-\rho^2}}\right)}{1 - \Phi\left(-\frac{\eta_1 + \rho\, \Phi^{-1}(u_2)}{\sqrt{1-\rho^2}}\right)}$$